OPERATION OF A MAGNETIC CUMULATION GENERATOR INTO A CAPACITATIVE LOAD

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Magnetic cumulation generators are powerful pulsed electrical-energy sources [1-3]. They substantially exceed capacitors as regards the stored energy density, and therefore capacitor banks are used in conjunction with magnetic cumulation generators (MCG) as a source of the initial energy. However, capacitors of higher energy capacity are now known. Pulse high-voltage capacitors with liquid insulators can store up to about 0.2 J/cm^3 [4], and molecular capacitors give even higher figures [5]. On the other hand, the high energy density in the MCG is observed only in the final volume, which is a small part of the whole generator. The design density of an MCG (ratio of the output energy to volume of the whole device) is about 10-50 J/cm^3. With this ratio of the energy densities, the advantage of an MCG is not so striking, which makes it desirable to include capacitors in the load circuit of the MCG, which opens up a new way of solving physics problems. For example, it has been suggested [6] that an MCG should be used as the charging device for fast capacitor accumulators in order to generate short pulses to supply linear induction electron accelerators, and shunting with a load capacitance is also used [7], this being fed from an inductive accumulator, which can also be employed when an MCG is used, etc. All of this makes it important to consider the operation of an MCG into a capacitative load, which has not previously been considered in the literature.

Here we consider very simple MCG schemes with capacitors: an MCG with a series RLC circuit and a transformer MCG with a capacitor in the secondary circuit with certain particular cases of the inductance-variation law. We assume that at the start a current I_0 flows in the MCG (subscript 0 denotes the initial value of the corresponding quantity), while the potential difference across the capacitor is $U_0 = 0$. The process is considered only during the operation of the MCG, since known relationships apply in the passive decay of the currents.

1. The decreasing inductance substantially alters the relationships in the series RLC circuit, because energy is generated in the circuit itself. An oscillatory state is also possible here, and the amplitude of the oscillations will increase if the resistance R is small. The case of very large R is not so interesting, because there is little effect on the variable circuit inductance L. Figure 1 shows the equivalent circuit of an MCG in a series tuned circuit. Here $L = L_g + L_l$, where L_g is the variable inductance of the MCG, L_l is the load inductance, and R is the circuit resistance, which formally includes all the magnetic-flux losses, while C is the load capacitance. The circuit current is described by

$$L\ddot{I} + (2\dot{L} + R)\dot{I} + (\ddot{L} + \dot{R} + 1/C)I = 0, \quad (1.1)$$

where a dot denotes differentiation with respect to time, and the voltage on the capacitor is

$$L\ddot{U} + (\dot{L} + R)\dot{U} + U/C = 0. \quad (1.2)$$

The L(t) and R(t) relationships are determined by the design of the MCG. We consider the solution to (1.1) and (1.2) for the case R = 0, $L(t) = L_0 \exp(-at)$, where a is a positive constant with the dimensions sec^{-1}. This L(t) is close to the law followed by the inductance in an MCG with a spiral of variable pitch. Then (1.1) is written as

$$\ddot{I} - 2a\dot{I} + a^2(1 + \theta^2 e^{at})I = 0,$$

where $\theta = 1/(a\sqrt{L_0 C})$, and the solution to the equation is

$$I = [C_1 J_0(x) + C_2 N_0(x)] \exp(at),$$

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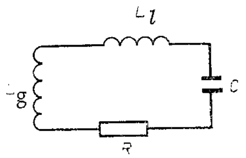

Fig. 1

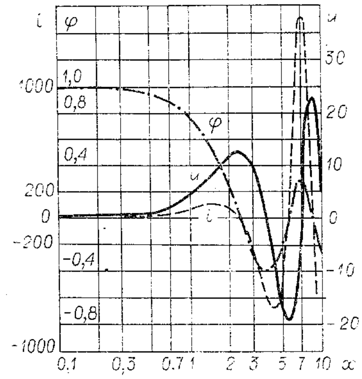

Fig. 2

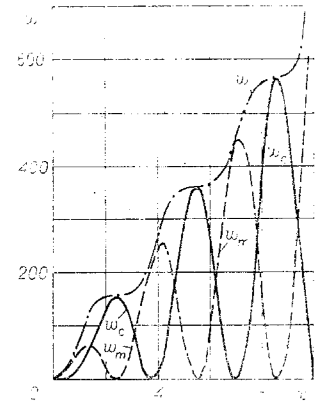

Fig. 3

where $J_0(x)$, $N_0(x)$ are Bessel functions of the first and second kinds, respectively, of zero order, whose argument is $x = 2\theta \exp(at/2)$; we have $x_0 = 2\theta$ for $t = 0$, and subsequently x increases, where $x/x_0 = \sqrt{\lambda}$, with $\lambda = L_0/L$.

When we have determined the constants of integration C_1 and C_2 from the initial conditions we get

$$i = [-N_1(x_0)J_0(x) + J_1(x_0)N_0(x)]\pi x^2/2x_0, \tag{1.3}$$

where $i = I/I_0$; from this we can get $i(t)$ because $t = 2a^{-1}\ln(x/x_0)$, and similarly we have

$$u = [-N_1(x_0)J_1(x) + J_1(x_0)N_1(x)]\pi x/2, \tag{1.4}$$

where $u = U/(I_0\sqrt{L_0/C})$.

The functions $J_1(x_0)$, $N_1(x_0)$ define the initial phase of the process; we denote the zeros of J_m and N_m, respectively, by μ_{mn} and η_{mn}, where m is the order of the function and n is the order of the zero. For $x < \eta_{01}$ the function $N_0(x)$ is monotonic, and therefore $i(x)$ is aperiodic, whereas it is oscillatory when x becomes greater than μ_{01}. The corresponding criteria for $u(x)$ are η_{11} and μ_{11}. The character of the process at the start of MCG operation is determined by x_0. If the operating time T of the MCG is sufficiently large, the initial aperiodic process is replaced by an oscillatory one with an exponential law for the inductance.

If x is small, one can restrict oneself to the first terms in the series expansion of the Bessel functions. The error is less than 1% for $x < 0.1$, while it is a few percent for $x < 0.3$. For small x_0

$$i \approx \lambda\left[J_0(x) + \frac{\pi x_0^2}{4} N_0(x)\right], \quad u \approx \sqrt{\lambda}\left[J_1(x) + \frac{\pi x_0^2}{4} N_1(x)\right].$$

If x_f also remains small (subscript f denotes the value of the corresponding quantity at the finish of operation of the MCG), then

$$I \approx \Phi_0/L, \; i \approx \exp(at) = \lambda, \; u \approx x_0(\lambda - 1)/2.$$

In that case the magnetic flux in the circuit persists and the current is independent of the capacitance.

For the oscillatory state we can obtain an approximate solution that is the more accurate the larger x:

$$i \approx \sqrt{\lambda}[-N_1(x_0)\cos(x - \pi/4) + J_1(x_0)\sin(x - \pi/4)],$$
$$u \approx [-N_1(x_0)\sin(x - \pi/4) - J_1(x_0)\cos(x - \pi/4)].$$

If the value of x_0 is also large, then $i \approx \lambda^{3/4}\cos(x - x_0)$, $u \approx \lambda^{1/4}\sin(x - x_0)$, i.e., there will be current and voltage oscillations increasing in amplitude and frequency. The final values i_f and u_f will be dependent on the phase of the oscillation at $t = T$.

Figure 2 shows $i(x)$ and $u(x)$ calculated from (1.3) and (1.4) for $x_0 = 0.1$, $x_f = 10$; at the start the process is aperiodic but it then becomes oscillatory. We also show $\varphi(x)$, the coefficient for magnetic-flux retention $\varphi = i/\lambda$. We have $\varphi \approx 1$ in the aperiodic state, while $\varphi \approx \lambda^{-1/4}\cos(x - x_0)$ in the oscillatory one, i.e., the flux oscillates in phase with the current with an amplitude decreasing in proportion to $\lambda^{1/4}$. Therefore, on working into a capacitance the magnetic flux in the generator may not be preserved even in the absence of a resistance.

We denote by W_c the energy in the capacitor and by W_m the energy in the inductance, and we have $w_c = u^2$, $w_m = i^2/\lambda$, $w = w_c + w_m$, where $w_c = W_c/W_0$; $w_m = W_m/W_0$; $w = (W_c + W_m)/W_0$; the $w_c(x)$, $w_m(x)$, $w(x)$

relationships have been calculated for the case $x_0 = 0.1$, $x_f = 10$ (Fig. 3). In the aperiodic state, with small x_f, the energy is stored in the main in the inductance, $w_m \gg w_c$. After the end of MCG operation, it is subsequently localized in the capacitance, but in that case the period $2\pi\sqrt{L_f C} \gg T$, while if we take a value approximately equal to T there is appreciable magnetic flux nonconservation.

2. Now let $R = 0$, $L = L_0(1 - at)$; this law for inductance is characteristic for example of coaxial MCG, MCG with a spiral of constant pitch, etc. Then (1.1) will have the form

$$(1 - at)\ddot{I} - 2a\dot{I} + a^2\theta^2 I = 0,$$

and the solution is

$$I = [C_1 J_1(y) + C_2 N_1(y)](1 - at)^{-1/2},$$

where the argument of the Bessel functions is $y = 2\theta(1 - at)^{1/2}$ and differs from x in decreasing with the passage of time. The I(y) dependence is inversely proportional to time, since $t = [1 - (y/y_0)^2]/a$, but it is directly proportional to L. On determining C_1 and C_2 we get

$$i = [N_0(y_0) J_1(y) - J_0(y_0) N_1(y)]\, \pi y_0^2/2y.$$

Similarly we can get

$$u = [N_0(y_0) J_0(y) - J_0(y_0) N_0(y)]\pi y_0/2.$$

When $y \lesssim 0.3$,

$$i \approx [(\pi y_0/2)^2 N_0(y_0) + \lambda J_0(y_0)],$$
$$u \approx [(\pi y_0/2) N_0(y_0) - y_0(0.577 + \ln (y/2)) J_0(y_0)].$$

In that case i_f and u_f are dependent on the initial phases of $N_0(y_0)$ and $J_0(y_0)$; if $J_0(y_0) \neq 0$, then $i \to \infty$ for $y \to 0$. If on the other hand $y_0 = \mu_{0n}$, the current is finite even for $y_f = 0$:

$$i_f = [N_0(\mu_{0n})](\pi\mu_{0n}/2)^2.$$

If y_0 is also small, then

$$I \approx \Phi_0/L, \; i \approx \lambda, \; u \approx y_0 \ln (y_0/y).$$

In the oscillatory state for large y_f

$$i \approx \lambda^{3/4} \cos (y_0 - y), \; u \approx \lambda^{1/4} \sin (y_0 - y).$$

Figure 4 shows u(y) for $y_f = 0.1$, $y_0 = \eta_{03}$, and $y_0 = \mu_{03}$; in the second case there is no increase in u for $y \to 0$.

For small y

$$\varphi \approx [(\pi y/2)^2 N_0(y_0) + J_0(y_0)].$$

Also, φ increases with y, and $\varphi \to J_0(y_0)$ for $y \to 0$. If y_0 is small, then $\varphi \approx 1$, and in the oscillatory state $\varphi \approx \lambda^{-1/4} \cos (y_0 - y)$.

Therefore, a uniform law for the inductance differs from the exponential case in that an initially aperiodic process at the start of MCG operation cannot be replaced by an oscillatory one, while the process oscillatory for a sufficiently large T is replaced by an aperiodic one at the end of MCG operation.

3. If $R \neq 0$ (1.1), (1.2) can be solved for a series of particular cases.

Let R be constant at $L_1 = L_0(1 - at)$; then (1.1) is written as

$$(1 - at)\ddot{I} + a(\nu - 2)\dot{I} + a^2\theta^2 I = 0,$$

where $\nu = R/aL_0$; incorporating the initial conditions we get

$$i = [N_\nu(y_0) J_{\nu-1}(y) - J_\nu(y_0) N_{\nu-1}(y)](\pi y_0/2)(y/y_0)^{\nu-1}.$$

The index of the Bessel function is here dependent on R, and therefore it is necessary to use the ν and y dependences of J_ν and N_ν [8]. Similarly,

$$u = [N_\nu(y_0) J_\nu(y) - J_\nu(y_0) N_\nu(y)](\pi y_0/2)(y/y_0)^\nu.$$

Then $\nu = 0$ for $R = 0$ and $\nu = 1$ for $R = -\dot{L}$, and in this range the transition to the aperiodic state as a function of ν will lie at a point between η_{01} and μ_{11}. For $\nu > 1$ there is no increase in the current amplitude,

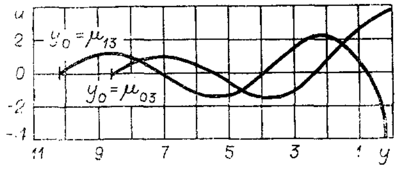

Fig. 4

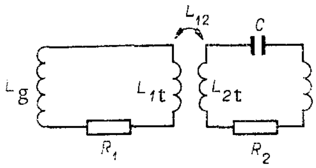

Fig. 5

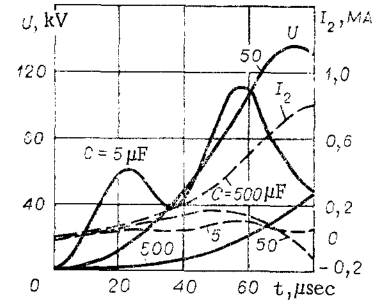

Fig. 6

and the position to the aperiodic state as ν increases shifts to larger y. For the oscillatory state ($y \gg 1$, $y \gg \nu$)

$$i \approx \lambda^{(3-2\nu)/4} \cos (y_0 - y), \quad u \approx \lambda^{(1-2\nu)/4} \sin (y_0 - y).$$

Clearly, the amplitude of I increases if $\nu < 3/2$ and the amplitude of U for $\nu < 1/2$.

We now assume that $L = L_0 \exp(-at)$ with constant R/L. Then from (1.1) we have

$$\ddot{I} + a(\nu - 2)\dot{I} + (a^2\theta^2 e^{at} - \nu + 1)I = 0,$$

and the solution is

$$I = [C_1 J_\nu(x) + C_2 N_\nu(x)]x^{-\nu/2},$$

where $\nu = R/aL$. With the given initial conditions

$$C_1 = I_0[AN_\nu(x_0) - BN_{\nu+1}(x_0)], \quad C_2 = I_0[AJ_\nu(x_0) + BJ_{\nu+1}(x_0)],$$

where A and B are constants expressed as fairly cumbersome polynomials dependent on ν and x_0. For large x_0

$$i \approx [A \cos (x - x_0) + B \sin (x - x_0)](2/\pi)^{1/2} x^{(1-\nu)/2}.$$

This relationship is somewhat simpler for other initial conditions.

In numerical solution of (1.1) and (1.2), the value of I_0 should be taken as the maximum possible for a given generator in order to make the fullest use of the performance of the MCG, no matter what the mode of creation in the circuit. If we reduce C at constant I_0, there is an increase in the amplitude of the first wave in U, while the subsequent rise in the amplitude of U during the MCG operation may be less important, and this state in fact represents a discontinuity in the initial current.

The frequency $(LC)^{-1/2}$ of the oscillations in the tuned circuit may exceed the equivalent frequency characteristic of the MCG, which is determined in the main by the properties of the generator on operation into inductive and resistive loads. This increases the generator resistance and causes additional energy losses. It would seem that special MCG designs are required to work with capacitative loads.

4. Figure 5 shows the equivalent circuit of an MCG with a capacitor in the secondary circuit of the transformer, where L_1, L_2, R_1, and R_2 are, respectively, the inductances and resistances of the primary and secondary circuits, with $L_1 = L_g + L_{1t}$, where L_g is the working inductance of the MCG, L_{1t} is the inductance of the primary winding, $L_2 = L_{2t} + L_l$, L_{2t} is the inductance of the secondary winding, L_{12} is the mutual inductance, and I_1 and I_2 are, respectively, the currents in the primary and secondary windings, which are defined by

$$\begin{aligned} L_1\dot{I}_1 + (\dot{L}_1 + R_1)I_1 + L_{12}\dot{I}_2 &= 0, \\ L_2\ddot{I}_2 + R_2\dot{I}_2 + L_{12}\ddot{I}_1 + I_2/C &= 0. \end{aligned} \tag{4.1}$$

With R_1, $R_2 = 0$, $U_0 = 0$, $I_{20} = 0$ we have $I_1 = \Phi_0/L_1 - L_{12}I_2/L_1$; the second term becomes comparable in magnitude with the first only at the end of MCG operation, so I_2 has little effect on the MCG operation even when there are high-frequency oscillations in the secondary circuit; I_2 is the sum of the MCG current from the inductance L_2 in the secondary circuit in the absence of the capacitance and of the current oscillating at the start of operation with frequency $\sim(L_2C)^{-1/2}$, which then falls. For $T \ll (L_eC)^{1/2}$, where $L_e = (L_1 - L_{12}^2/L_2)L_2^2/L_{12}^2$, there are no oscillations in the secondary circuit.

If $L_1 = L_0/(1 + at)$ (this is not so characteristic of an MCG but fits fairly closely to the inductance law for a sectioned spiral), then (4.1) can be solved analytically with the above assumptions:

$$i_2 = [N_1(z_0)J_1(z) - J_1(z_0)N_1(z)]\pi z_0 l(1 + \alpha)/(k^2 z),$$

where $i_2 = -I_2L_2/L_{12}I_{10}$, $l = L_0/L_{1t}$, $\alpha = L_l/L_{2t}$, k is the transformer coupling coefficient, I_{10}, I_{20}, L_0 are the initial values of I_1, I_2, L_1, $\theta = 1/a\sqrt{L_2C}$, and

$$z = [2l/(\theta k^2)](1 + \alpha)^{1/2}[1 + \alpha - k^2(1 + at)/l]^{1/2}.$$

As the MCG operates, z decreases from $z_0 = (2l/\theta k^2)\sqrt{[(1+\alpha)(1+\alpha-k^2/l)]}$ to $z_f = [2l/(\theta k^2)][(1+\alpha)(1+\alpha-k^2)]^{1/2}$; if $z_f > \mu_{11}$, then i_2 has an oscillatory component for large z_f:

$$i_2 \approx [2l(1+\alpha)z_0^{1/2}/(k^2z^{3/2})]\sin(z_0 - z).$$

For $z_0 < \eta_{11}$ we have $i_2 \approx (1 - z_0^2/z^2)l(1+\alpha)/k^2$, i.e., the current is as in the case of the MCG working in the absence of a capacitor in the secondary circuit.

We integrate the expression for i_2 to get $u = F/\theta$, where $u = -U(L_2C)^{1/2}/(I_{10}L_2)$; $F = [N_1(z_0)J_0(z) - J_1(z) \times N_0(z)](\pi z_0/2) + 1$; for large z_f we have $u \approx [(z_0/z)^{1/2}\cos(z_0 - z) - 1]/\theta$, while for small z_0 we have $u \approx (z_0 - z)^2/2\theta z$.

Then $w_c = u^2k^2/[l(1+\alpha)]$, $w_m = i_2^2\alpha k^2/[l(1+\alpha)^2]$.

Figure 6 shows $I_2(t)$ and $U(t)$ as calculated for the K-160 transformer generator [9] loaded with various capacitors. The secondary circuit of the generator is closed 80 μsec before the end of operation, and in this section $L_1(t) \approx 91\cdot10^{-9} - 10.6\cdot10^4t - 1.95t^2 + 6.2\cdot10^4t^3$, $R_1(t) \approx 8.8\cdot10^{-5} - 0.83t$. The secondary winding has 16 turns, $L_{1t} = 26$ nH, $L_{2t} = 6.8$ μH, $L_{12} = 0.4$ μH, $k = 0.96$, $\alpha = 0.44$, $R_2 = 2\cdot10^{-2}$ Ω, $I_{10} = 5.4$ MA.

These examples show that an MCG can work efficiently into a capacitative load under certain conditions. The character of the MCG operation then differs substantially from that with inductive or resistive loads. The main difference is due to the possibility of an oscillatory component in the current. Even in the absence of resistances, the magnetic flux in the MCG circuit may not be retained.

The output energy of the MCG increases less rapidly in the oscillatory state, but in the aperiodic one it is localized in the main only in the inductance. The capacitance can also be connected in other ways, e.g.: a capacitor in parallel with an inductive or resistive load, with current-switching components in the circuit, etc.

Specially designed MCG are best for use with capacitative loads. The use of such an MCG would enable one, for example, to obtain several equal current half-waves from a capacitor battery. The design of an open tuned circuit supplied from an MCG allows one to transform the MCG energy into radio waves. Considerable interest also attaches to operation of an MCG into a line with distributed parameters.

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